# Effects of polarization on inverse Bremsstrahlung heating of a plasma

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A molecular dynamic (MD) code is used to compare the rates of heating by inverse Bremsstrahlung (IB) for circularly and linearly polarized radiation. For low intensities the heating rate is found to be independent of polarization. However, at higher intensities the variation of the heating rate with the radiation intensity is found to exhibit a sharper peak for circularly polarized than linearly polarized radiation. This difference is explained in terms of differences in the variation of the electron quiver speed during the optical cycle for linearly and circularly polarized radiation. An analytical expression—which includes a term which is nonlinear in the density of the plasma—for the rate of IB heating is fitted to the rates calculated by the MD code.

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## I. INTRODUCTION

In an earlier paper [1] we described a molecular dynamic (MD) code for calculation of the rate of inverse Bremsstrahlung (IB) heating in a plasma. As discussed in detail in that earlier work, there are several advantages in applying the MD approach to calculations of the IB heating rate. Most importantly, the equations solved are fundamental and so avoid several assumptions which are inherent to alternative methods. For example, in methods employing a Coulomb logarithm there is always a degree of arbitrariness in choosing the inner and outer cutoffs. The MD code is also capable of handling many-body collisions correctly, and is therefore not restricted by the assumptions inherent to a two-particle collision approach. It has been found [1] that this results in the heating rate varying nonlinearly with the plasma density. Finally, the MD approach is not restricted to weakly coupled plasmas. The main disadvantage of the MD approach is that it is much slower than calculations based on classical, statistically based methods.

Many calculations of the rate of IB heating have been presented, some of the more important being Schlessinger and Wright [2] in 1979, Polishchuk and Meyer-Ter-Vehn [3] in 1994 and Pert [4] in 1995. However, to our knowledge, the dependance of the heating rate on the polarization of the incident radiation has not been investigated despite the fact that these earlier methods could all, in principle, be adapted to calculate the heating rate in this case. Kostyukov and Rax [5] have presented calculations of the rate of IB heating for circularly polarized radiation, but their work focuses on relativistic effects in the ultrahigh intensity regime and does not discuss explicitly the relation between the circular and the linear polarized cases. Pert [6] has done calculations on elliptically polarized radiation using a Fokker-Planck code. The results presented by Pert agree qualitatively with the results presented in this paper.

In the present paper we use an MD code to compare directly the rates of IB heating for circularly- and linearly-polarized radiation. For low intensities, where the electron quiver velocity is of the order of the thermal velocity or less, the heating rate is found to be the same for the two polarizations. However, at higher intensities, where the electron quiver velocity is much greater than the thermal velocity, the variation of the heating rate with the radiation intensity is found to exhibit a sharper peak for circularly polarized than linearly polarized radiation. This difference is explained in terms of differences in the variation of the electron quiver speed during the optical cycle for linearly and circularly polarized radiation. An analytical expression for the rate of IB heating is fitted to the rates calculated by the MD code; it is found that it is necessary to include a term which is nonlinear in the density of the plasma, as found previously [1] for the case of linearly polarized radiation.

## II. THE MOLECUAR DYNAMIC CALCULATION

The operation of the MD code has been described in detail previously [1], and hence here we give only a short description of its main features.

A molecular dynamic calculation differs from statistically based methods in that one starts by considering the force acting on a given particle by all the other particles in the system, rather than considering only two-particle collisions. However, solving the individual force equations for each of the $N$ particles in a plasma would require $N^2/2$ individual force calculations to be undertaken every integration timestep. Further, the time resolution has to be very high to ensure that even the hardest collisions are treated correctly and hence direct integration of all the terms in the force equations is too slow to be realistic—a calculation involving $2\times10^4$ particles would take several months. A large reduction in the number of calculations required can be achieved by employing the particle-particle-particle-mesh ($P^3M$) method first described by Hockney and Eastwood [7]. In this method the force on a particle is divided into a collective, long-range term from the majority of the particles (particle-mesh); and a short range term arising from particles close to the particle in question (particle-particle). Using this general idea, we calculate the long range effects by solving the Poisson equation on a mesh, and treating the short-range effects by direct integration of the Coulomb force.

## III. CALCULATIONS

Most important to every molecular dynamic calculation is the right choice of the initial distribution of particles over velocity and position. This is slightly problematic since there is no analytical solution for the phase-space distribution of electrons and ions for plasmas which are not weakly coupled. Whilst the density of the plasma is homogenous on a large scale, it certainly is not on a small scale; the well known effect of Debye shielding, for example, leads to local inhomogeneity as described by the two-particle correlation function. These, and higher order effects, will play a role in moderately or strongly coupled plasmas. For a plasma with a given mean density and temperature, the initial distribution over velocity and position was calculated by running the MD code—in the absence of any applied radiation field—until a steady state was reached [1].

The heating rate was determined by running the calculations for 4 cycles of the laser field, calculating the increase in the mean energy of the plasma electrons, and hence deducing the rate of increase of electron temperature. Since the rate of IB heating is relatively insensitive to the temperature of the plasma, the small change in temperature $\Delta T_e$ is accurately given by $\Delta T_e = R\Delta t$, where $R$ is the heating rate at the initial plasma temperature, and $\Delta t$ is the interval over which the heating is calculated. To quantify this, using the analytical fit discussed below [Eq. (2)], a plasma with $n_e = 10^{20}$ cm$^{-3}$ and initial temperature $T_e = 10$ eV is heated to 14.37 eV over the 14.14 fs of the simlation. The relative difference for the heating rate between a 10 eV plasma ($R = 0.309$ eV/fs) and a 14.37 eV plasma ($R = 0.301$ eV/fs) as predicted by Eq. (2) is 2.6%, well within the expected error.

As discussed previously [1], the statistical and numerical errors in the calculated rate of IB heating are estimated to be approximatley 5%. To ensure that differences in the calculated rates of IB heating for the two polarization cases considered are not due to differences in the initial phase-space distribution or due to different choice of time-step or inner cutoff, for a given plasma density and temperature the calculated rates of IB heating were performed with the same parameters and initial distribution.

For the case of linearly polarized radiation the electric field of the incident radiation is assumed to be of the form $\mathbf{E}(t) = E_0 \cos(\omega t)\mathbf{e}_x$, where the amplitude of the electric field is related to the intensity $I$ of the radiation by $E_0 = \sqrt{2Z_0 I}$, in which $Z_0$ is the impedance of free space. In the steady-state the velocity of the center of mass of the electron distribution is given by $\mathbf{v}_{\mathrm{c.m.}}(t) = v_E \sin(\omega t)\mathbf{e}_x$, where $v_E = eE_0/m_e\omega$. Hence, at $t=0$ the velocity of the center of mass is zero.

For circularly polarized radiation of the same intensity $I$ the electric field is described by $\mathbf{E}(t) = (E_0/\sqrt{2})[\cos(\omega t)\mathbf{e}_x + \sin(\omega t)\mathbf{e}_y]$, and the motion of the center of mass of the electron distribution by $\mathbf{v}_{\mathrm{c.m.}}(t) = (v_E/\sqrt{2})[\sin(\omega t)\mathbf{e}_x - \cos(\omega t)\mathbf{e}_y]$. Hence at $t=0$ the initial velocity distribution of the electrons needs to be offset by $-(v_E/\sqrt{2})\mathbf{e}_y$.

### Results

Figure 1 shows, for several different initial plasma conditions, the variation with intensity of the calculated rates of IB heating for both linearly and circularly polarized radiation.

## IV. DISCUSSION

The results presented in Fig. 1 show that at low intensities the rates of IB heating are essentially independent of the polarization of the radiation. However, for higher intensities there is a clear difference between the heating rates: for circularly polarized radiation the peak heating rates are higher, whilst at the highest intensities the heating rates are lower. It is worth emphasizing that the same initial distribution and code parameters were used for the two polarization cases; as such the observed differences in heating rates arise from the differences in polarization alone, and are not numerical or statistical fluctuations.

The differences in behavior between the two polarization cases can be explained as follows. For a given set of plasma conditions there exists an electron quiver speed $v_E$ that optimizes the rate of IB heating. This optimum arises from the fact that, within a simplified two-particle picture, the instantaneous rate of IB heating is proportional to the product of the quiver energy of the electron and the momentum transfer cross section. The cross section for momentum transfer depends on the relative velocity $v_r$ of the colliding electron and ion, and varies approximately as $v_r^{-1/4}$. At low radiation intensities the relative electron-ion velocity is dominated by the thermal speed of the electron, and hence the rate of IB heating increases with the quiver energy, and hence with the radiation intensity. At high intensities, however, the relative electron-ion velocity is dominated by the electron quiver velocity leading to a decrease in the rate of IB heating despite the increasing electron quiver energy.

At low intensities, therefore, the rate of heating depends only on the electron quiver energy, and since the cycle-averaged quiver energy is independent of polarization, the rate of IB heating is also polarization independent.

Differences in the rate of IB heating for the two polarizations occur at higher intensities since the relative electron-ion velocity then depends on the quiver speed. These differences occur despite the fact that the mean quiver speed is independent of the polarization since for circularly polarized radiation the quiver *speed* remains constant throughout the optical cycle, whilst for linearly polarized radiation the quiver speed varies from zero to $v_E$. Hence for circularly polarized radiation the rate of IB heating is constant throughout the optical cycle, whilst for linearly polarized radiation the heating rate varies. As a consequence, the relatively sharp peak in the variation of $R$ with intensity found for circular polarization is smoothed out for linear polarization.

For a given plasma density and temperature the instantaneous rate of IB heating depends only on the instantaneous quiver speed, and since this is constant for circularly polarized radiation the instantaneous heating rate is the same as the cycle-averaged rate calculated for circular polarization, $R^{\mathrm{circ}}(I)$. For linearly polarized radiation the cycle-averaged heating rate is then given by averaging this over the range of quiver energies during the optical cycle:

$$R^{\mathrm{linear}}(I) = \frac{1}{2\pi}\int_0^{2\pi} R^{\mathrm{circ}}(2I\cos^2 \omega t)\, d(\omega t). \tag{1}$$

It is clear that averaging over the optical cycle will lead to a smoother variation of $R^{\mathrm{lin}}(I)$ than found for $R^{\mathrm{circ}}(I)$.

The MD calculations presented above are useful in that they provide quantitative results that are free from approxi-

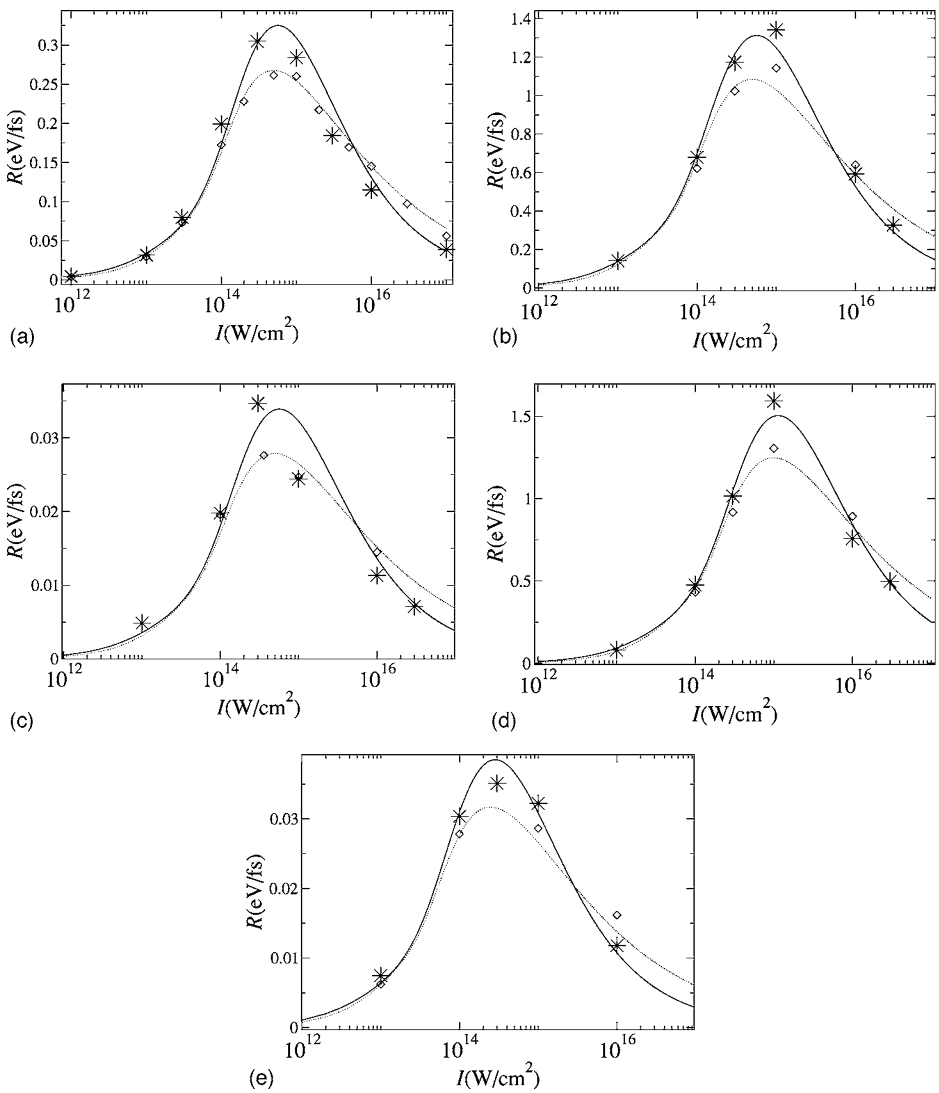


FIG. 1. Comparison of the heating rate $R$ for circularly polarized radiation (stars) and linearly polarized radiation (diamonds) in plasmas with initial values of (a) $n_e=10^{20}$ cm$^{-3}$, $T_e=10$ eV; (b) $n_e=5\times10^{20}$ cm$^{-3}$, $T_e=10$ eV; (c) $n_e=10^{19}$ cm$^{-3}$, $T_e=10$ eV; (d) $n_e=8\times10^{20}$ cm$^{-3}$, $T_e=20$ eV; (e) $n_e=10^{19}$ cm$^{-3}$, $T_e=5$ eV. Also shown are fitted analytical expressions for the heating rate for circularly (solid line) and linearly (dotted line) polarized radiation.

mations. However, they are slow to run and therefore impractical for calculations in which evaluation of the IB heating rate is but a small part. It is useful therefore to use the results of the MD calculation to find an analytical function for the heating rate. As described earlier for the case of linear polarization [1], a suitable analytical function is a modified version of the IB-heating formula derived by Polishchuk and Meyer-ter-Vehn [3]. Following the discussion above, despite the fact that Polishchuk and Meyer-ter-Vehn derived their expression for the case of linear polarization, the general behavior of the heating rate with laser intensity is expected to be similar for the case of circularly polarized radiation.

We therefore choose to fit the results of the MD calculation to the following function:

$$R=\frac{dT_e}{dt}=\frac{8e^4Z^2v_E^2}{3m(4\pi\epsilon_0)^2(v_E^2+v_e^2)^{3/2}}\alpha(n_i)\ln\Lambda, \tag{2}$$

with

$$\alpha(n_i)=n_i\left(1-\frac{n_i}{C_2}\right) \tag{3}$$

and

$$\ln\Lambda=C_1\frac{\ln\left[C_3\xi+C_4\xi^2+\exp\left(\frac{1}{3}\sqrt{\pi/2}\right)\right]}{C_5+[\ln(1+\xi)-C_6]^2}\times\ln\left[\exp(1)+\frac{T_e}{\hbar\omega}\right],$$

where

$$\xi=\frac{mv_E^2}{T_e}.$$

The modifications to the expression derived by Polishchuk and Meyer-ter-Vehn are as follows. The first factor in the heating rate $R$, $8e^4Z^2v_E^2/3m(4\pi\epsilon_0)^2(v_E^2+v_e^2)^{3/2}$, is derived from the heating rate for two-particle Coulomb collisions. We have introduced the second factor, $\alpha(n_i)$, to allow for nonlinear variation of the heating rate with plasma density arising from three or more body collisions. The third factor, the Coulomb logarithm, was modified from the expression derived by Polishchuk and Meyer-ter-Vehn to allow for the sharper peak in the variation of the IB heating rate with intensity which is observed for circularly polarized radiation, and stabilized for cases where $T_e \simeq \hbar\omega$ as discussed previously [1]. We note that Eq. (2) has a different form than that which we used previously to fit the IB heating rate for linear polarization [1]. The expression used for linearly polarized radiation did not fit the calculated heating rates for circular polarization very well; in particular the sharper peaks found for circular polarization were not well reproduced. It should be stressed that the analytical expressions we have used are modifications of those derived by Polishchuck and Meyer-ter-Vehn [3], but these modifications are not based on any physical model and were merely chosen to fit the results of the MD calculations over a wide range of conditions.

The constants $C_i$, $i=1$ to 6 were determined by numerically fitting Eq. (2) to all the calculated heating rates for circular polarization presented in Fig. 1. This yielded the following values for the fit parameters:

$$C_1 = 46.710,$$

$$C_2 = 2.180 \times 10^{27}\ \mathrm{m}^{-3},$$

$$C_3 = 0.0725,$$

$$C_4 = 0.102,$$

$$C_5 = 19.297,$$

$$C_6 = 2.933.$$

The resulting fits are shown in Fig. 1. We see that Eq. (2) fits the results of the MD calculations quite well, and certainly gives more accurate values for the heating rate than using our earlier expression [1] for linearly polarized radiation, or using functions derived by a two-body Coulomb logarithm approach. One should bear in mind, however, that Eq. (2) has been determined by fitting to a relatively low number of MD calculations performed in the density regime $n_i < 10^{21}\ \mathrm{cm}^{-3}$, intensity regime $10^{12}\ \mathrm{W\,cm}^{-2} < I < 10^{17}\ \mathrm{W\,cm}^{-2}$, and temperature regime $5\ \mathrm{eV} < T_e < 20\ \mathrm{eV}$. Caution is advised when using it outside this regime.

We now illustrate that the cycle-averaged heating rate for linearly polarized radiation may be deduced from the calculated rates for circular polarization. Figure 2 shows the fitted analytical functions for linearly and circularly polarized radiation for a plasma with $n_e = 10^{20}\ \mathrm{cm}^{-3}$ and $T_e = 10$ eV. Also shown is the heating rate for linearly polarized radiation calculated by integrating the fitted expression for circularly polarized radiation [Eq. (2)] over the optical cycle using Eq. (1). The agreement with the fitted analytical formula for linearly polarized radiation is very close; the small differences arise only from inaccuracies in the analytical formulas.

The nonlinear dependence of the heating rate on the plasma density, as introduced by the term $\alpha$ from Eq. (2), is

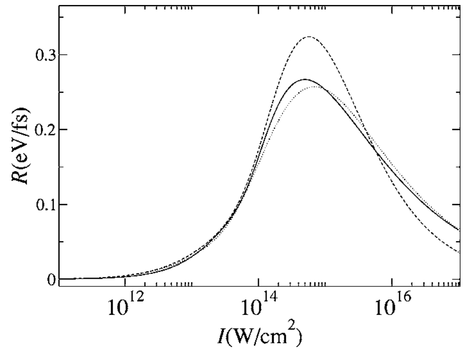


FIG. 2. Comparison of the heating rate $R$ using the fitted formulas for a plasma with initial values of $n_e = 10^{20}\ \mathrm{cm}^{-3}$, $T_e = 10$ eV. The three curves show the fit to the linear heating rates (solid line), the fit to the circular heating rates (dashed line), and the linear heating rate calculated by integrating over the circular curve (dotted line), Eq. (1).

consistent with the result found previously for linear polarization. The deduced value of $C_2 = 2.180 \times 10^{21}\ \mathrm{cm}^{-3}$ (compared to $C_2 = 2.211 \times 10^{21}\ \mathrm{cm}^{-3}$ for the linear case) shows that the heating will vary nonlinearly with plasma density for plasmas with densities greater than $\approx 5 \times 10^{20}\ \mathrm{cm}^{-3}$. The nonlinear dependence of the heating rate on plasma density is likely to be caused by the increasing importance of three- (and more) body collisions at high density.

## V. CONCLUSION

In summary, we have used an MD code to compare directly the rates of IB heating for circularly and linearly polarized radiation. For low intensities the heating rate was found to be the same for the two polarizations. However, at higher intensities the variation of the heating rate with the radiation intensity was found to differ for the two polarization cases. This difference was explained in terms of differences in the variation of the electron quiver speed during the optical cycle for linearly and circularly polarized radiation. An analytical expression for the rate of IB heating was fitted to the rates calculated by the MD code that could be incorporated into larger plasma codes. As found previously [1] for the case of linearly polarized radiation, the results suggest that the heating rate increases sublinearly with plasma density for densities greater than approximately $5 \times 10^{20}\ \mathrm{cm}^{-3}$.


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